LINEAR PROGRAMMING MODEL FOR OPTIMAL INVESTMENT ALLOCATION IN A MULTI-ASSET FUND

1. Overview

Client:

A mid-sized U.S.-based investment advisory firm managing diversified mutual fund portfolios with exposure to equities, bonds, and alternatives

Objective:

To construct a linear programming model that allocates capital across multiple asset classes to maximize expected return while meeting regulatory, sector, and risk exposure constraints.

2. Background

The fund used historical returns and qualitative scoring to determine asset allocations across 10 classes. However, this method lacked rigor and underperformed benchmarks in three of the past five years. We were brought in to introduce a mathematically optimized allocation strategy using LP to enhance return potential while satisfying diversification mandates.

3. Problem Definition

Business Goal:

Maximize the expected return of a \$20 million portfolio subject to asset limits, sector constraints, and minimum diversification requirements.

Constraints:

- Total investment = \$20 million
- Maximum allocation to any single asset class = 20%
- Minimum allocation to bonds = 30%
- At least 15% combined allocation to REITs and infrastructure
- Risk (Beta-weighted exposure) ≤ 1.0
- Non-negativity constraint for each investment amount

4. Data Summary (Simplified)

Asset Class	Expected Return (%)	Beta Score	Allocation Limit (%)
U.S. Equities	10.2	1.1	≤ 20
International Eq.	9.5	1.0	≤ 20
Corporate Bonds	5.2	0.6	≥ 10
Government Bonds	3.8	0.3	≥ 20
REITs	7.4	0.8	≥ 5
Infrastructure	6.9	0.7	≥ 10
Commodities	6.1	0.9	≤ 10
Cash	2.0	0.0	≥ 2

5. Methodology

Software Used:

- Excel Solver (Simplex LP) for quick deployment
- Python (PuLP library) for model validation
- Final documentation in Word with embedded visuals and source code appendix

Model Formulation:

Decision Variables:

Let x_i represent the amount invested in asset class i

Objective Function:

$$\text{Maximize } Z = \sum_{i=1}^{8} r_i \cdot x_i$$

Where r_i = expected return of asset i

Constraints:

1. Total budget:

$$\sum_{i=1}^{8} x_i = 20{,}000{,}000$$

Allocation limits and sector constraints:

1. $x_1 \leq 0.20 \times 20{,}000{,}000$
2. $x_3 + x_4 \geq 0.30 \times 20{,}000{,}000$
3. $x_5 + x_6 \geq 0.15 \times 20{,}000{,}000$

Risk exposure constraint:

$$\sum_{i=1}^{8} \beta_i \cdot \left(\frac{x_i}{20{,}000{,}000}\right) \leq 1.0$$

1. Non-negativity:

$$x_i \geq 0$$

6. Solution and Interpretation

Optimized Allocation (Rounded):

Asset Class	**Investment (\$)**	**Allocation (%)**
U.S. Equities	4,000,000	20.0
Int'l Equities	3,200,000	16.0
Corporate Bonds	2,800,000	14.0
Govt. Bonds	3,400,000	17.0
REITs	1,600,000	8.0
Infrastructure	1,600,000	8.0
Commodities	1,000,000	5.0
Cash	2,400,000	12.0

Model Output:

- Expected portfolio return: **7.96%**
- Benchmark return (previous allocation): **7.32%**
- Outperformance: **+8.7% annualized improvement** in net return

- Portfolio Beta: **0.97 (within limit)**

7. Visual Outputs Provided

- Pie chart: Optimized vs. actual portfolio distribution
- Line graph: Return vs. Risk profile of optimized allocation
- Stacked bar: Sector-level exposure changes pre/post model
- Sensitivity table: Change in return vs. changes in bond minimums

8. Deliverables

- Fully functional Excel Solver file with adjustable constraints
- Python script for advanced modeling scenarios
- 18-page written report including:
    - LP model formulation
    - Financial interpretation
    - Assumption documentation
    - Strategic recommendations

9. Impact & Outcome

Metric	Before Optimization	After LP Model	Change
Expected Return	7.32%	7.96%	+8.7%
Portfolio Beta	1.03	0.97	Reduced exposure
Compliance with Policy Caps	Partial	Fully compliant	↑ Risk control
Allocation Adjustments	Ad hoc	Model-based	↑ Precision

10. Recommendations

- Integrate LP modeling into quarterly portfolio rebalancing
- Adjust constraints dynamically based on market outlook or client mandates
- Use stochastic LP or robust optimization for modeling uncertain returns

- Train analyst team on using Python-based LP tools for greater flexibility

11. Strategic Value Delivered

- Introduced a **transparent, rules-based asset allocation strategy**
- Enhanced return potential while respecting firm-wide risk controls
- Enabled future **scenario testing and regulatory reporting** through reproducible models
- Positioned the fund for **more consistent, defensible investment decisions**